Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2012

Marking Scheme

Applied Mathematics

Ordinary Level

General Guidelines

1. Penalties of three types are applied to candidates' work as follows:

   Slips - numerical slips S(–1)

   Blunders - mathematical errors B(–3)

   Misreading - if not serious M(–1)

   Serious blunder or omission or misreading which oversimplifies:
   - award the attempt mark only.

   Attempt marks are awarded as follows: 5 (att 2), 10 (att 3).

2. The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. A car travels along a straight level road.
It passes a point P with a speed of $8\ \text{m s}^{-1}$ and accelerates uniformly for 12 seconds to a speed of $32\ \text{m s}^{-1}$.
It then travels at a constant speed of $32\ \text{m s}^{-1}$ for 7 seconds.
Finally the car decelerates uniformly from $32\ \text{m s}^{-1}$ to rest at a point Q.
The car travels 128 metres while decelerating.

Find **(i)** the acceleration

(ii) the deceleration

(iii) $|PQ|$, the distance from P to Q

(iv) the speed of the car when it is 72 m from Q.

(i)
$$v = u + ft$$
$$32 = 8 + f(12)$$
$$f = 2\ \text{m s}^{-2}$$
10

(ii)
$$v^2 = u^2 + 2fs$$
$$(0)^2 = (32)^2 + 2f(128)$$
$$f = -4\ \text{m s}^{-2}$$
10

(iii)
$$s = ut + \tfrac{1}{2}at^2$$
$$s_1 = 8(12) + \tfrac{1}{2}(2)(144)$$
$$s_1 = 240\ \text{m.}$$
10

$$s_2 = 32 \times 7$$
$$= 224\ \text{m}$$
5

$$s_3 = 128\ \text{m}$$

$$|PQ| = 240 + 224 + 128$$
$$= 592\ \text{m}$$
5

(iv)
$$v^2 = u^2 + 2fs$$
$$(0)^2 = u^2 + 2(-4)(72)$$

$$u = 24\ \text{m s}^{-1}$$
10

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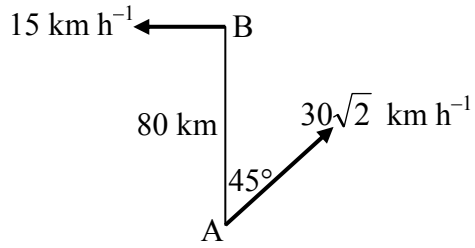
2. Ship A is positioned 80 km south of ship B. A is moving north-east at a constant speed of $30\sqrt{2}$ km h^{-1}.

B is moving due west at a constant speed of 15 km h^{-1}.



Find **(i)** the velocity of A in terms of $\vec{i}$ and $\vec{j}$

(ii) the velocity of B in terms of $\vec{i}$ and $\vec{j}$

(iii) the velocity of A relative to B in terms of $\vec{i}$ and $\vec{j}$

(iv) the shortest distance between A and B in the subsequent motion.

(i) $\vec{V}_A = 30\sqrt{2}\sin 45\ \vec{i} + 30\sqrt{2}\cos 45\ \vec{j}$

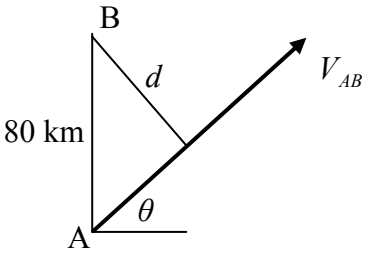
$= 30\ \vec{i} + 30\ \vec{j}$ — 10

(ii) $\vec{V}_B = -15\ \vec{i} + 0\ \vec{j}$ — 10

(iii) $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

$= \left(30\ \vec{i} + 30\ \vec{j}\right) - \left(-15\ \vec{i} + 0\ \vec{j}\right)$

$= 45\ \vec{i} + 30\ \vec{j}$ — 10



(iv) $\theta = \tan^{-1}\left(\frac{30}{45}\right)$

$= 33{\cdot}69°$ — 10

$d = 80\cos 33{\cdot}69$

$= 66{\cdot}56$ km — 10

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3. **(a)** A ball is kicked from a point P on horizontal ground with a speed of $20\ \text{m s}^{-1}$ at 45° to the horizontal.

The ball strikes the ground at Q.

Find **(i)** the time it takes the ball to travel from P to Q

(ii) $|PQ|$, the distance from P to Q.

(i) $s_y = ut + \frac{1}{2}at^2$

$0 = 20\sin 45 \times t - 5t^2$

$t = 2\sqrt{2}$ s

(ii) $s_x = ut + \frac{1}{2}at^2$

$|PQ| = 20\cos 45 \times t + 0$

$= 20 \times \frac{1}{\sqrt{2}} \times 2\sqrt{2}$

$= 40$ m

10

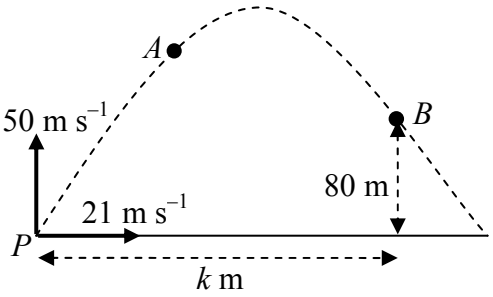
10

20

3 **(b)** A particle is projected with initial velocity $21\,\vec{i} + 50\,\vec{j}$ m s^{-1} from point P on a horizontal plane.

A and B are two points on the trajectory (path) of the particle.

The particle reaches point A after 3 seconds of motion.



The displacement of point B from P is $k\,\vec{i} + 80\,\vec{j}$ metres.

Find **(i)** the velocity of the particle at A in terms of $\vec{i}$ and $\vec{j}$

(ii) the speed and direction of the particle at A

(iii) the value of k.

(i)

$$v = u + at$$
$$v_x = 21 + 0$$
$$= 21$$

5

$$v_y = 50 - 10 \times 3$$
$$= 20$$

$$v = 21\,\vec{i} + 20\,\vec{j}$$

5

(ii)

$$|v| = \sqrt{21^2 + 20^2}$$
$$= 29 \text{ m s}^{-1}$$

5

$$\alpha = \tan^{-1}\left(\frac{20}{21}\right)$$
$$= 43{\cdot}6°$$

5

(iii)

$$80 = 50t - 5t^2$$
$$t^2 - 10t + 16 = 0$$
$$(t-2)(t-8) = 0$$
$$t = 8$$

5

$$s_x = ut + \tfrac{1}{2}at^2$$
$$k = 21 \times 8$$
$$= 168$$

5

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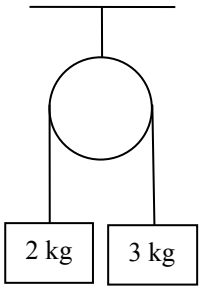
4. **(a)** Two particles of masses 2 kg and 3 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley.



The system is released from rest.

Find **(i)** the common acceleration of the particles

(ii) the tension in the string.

(i)

$3g - T = 3a$

$T - 2g = 2a$

$g = 5a$

$a = \frac{g}{5} = 2 \text{ m s}^{-2}$

(ii)

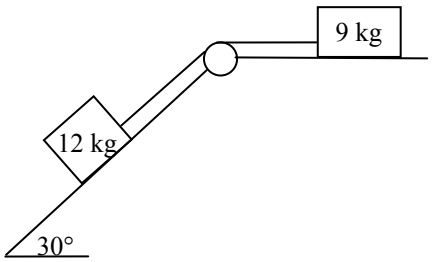
$T = 2g + 2a$

$= 20 + 4$

$= 24 \text{ N}$

5 5 5 5 20

4 **(b)** Masses of 9 kg and 12 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley as shown in the diagram.



The 9 kg mass lies on a rough horizontal plane and the coefficient of friction between the 9 kg mass and the plane is $\frac{1}{3}$.

The 12 kg mass lies on a smooth plane which is inclined at 30° to the horizontal.

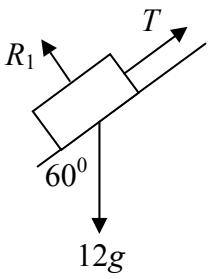
The system is released from rest.

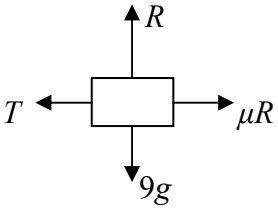
(i) Show on separate diagrams the forces acting on each particle.

(ii) Find the common acceleration of the masses.

(iii) Find the tension in the string.

(i)





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(ii)

$$12g\cos 60 - T = 12a$$ 5

$$T - \mu R = 9a$$ 5

$$6g - 3g = 21a$$

$$a = \frac{10}{7} = 1{\cdot}4 \text{ m s}^{-2}$$ 5

(iii)

$$T = 3g + 9a$$

$$= \frac{300}{7} = 42{\cdot}9 \text{ N}$$ 5

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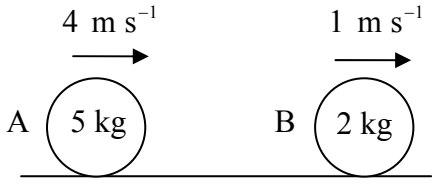
5. A smooth sphere A, of mass 5 kg, collides directly with another smooth sphere B, of mass 2 kg, on a smooth horizontal table.



A and B are moving in the same direction with speeds of $4\ \text{m s}^{-1}$ and $1\ \text{m s}^{-1}$ respectively.

The coefficient of restitution for the collision is $\frac{1}{6}$.

Find **(i)** the speed of A and the speed of B after the collision

(ii) the loss in kinetic energy due to the collision

(iii) the magnitude of the impulse imparted to A due to the collision.

(i)
$$5(4)+2(1) = 5v_1+2(v_2)$$
$$22 = 5v_1+2v_2$$

10

$$v_1 - v_2 = -e(4-1)$$
$$= -\frac{1}{6}(3)$$
$$= -\frac{1}{2}$$

10

$$v_1 = 3\ \text{m s}^{-1} \text{ and } v_2 = \frac{7}{2}\ \text{m s}^{-1}$$

5

(ii)
$$\text{KE}_b = \tfrac{1}{2}(5)(4)^2 + \tfrac{1}{2}(2)(1)^2$$
$$= 41$$

5

$$\text{KE}_a = \tfrac{1}{2}(5)(3)^2 + \tfrac{1}{2}(2)(3{\cdot}5)^2$$
$$= 34{\cdot}75$$

5

$$\text{KE}_b - \text{KE}_a = 41-34{\cdot}75$$
$$= 6{\cdot}25\ \text{J}$$

5

(iii)
$$\text{Impulse} = \left|(5)(3) - (5)(4)\right|$$
$$= 5\ \text{N s}$$

10

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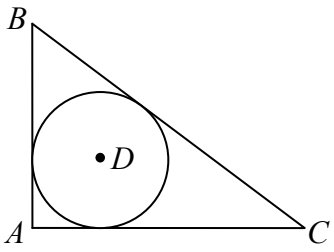
6. **(a)** Particles of weight 4 N, 7 N, 3 N and 5 N are placed at the points $(p,\ 2)$, $(-6,\ 1)$, $(9, q)$ and $(12, 13)$, respectively.
The co-ordinates of the centre of gravity of the system are $(p,\ q)$.

Find **(i)** the value of p

(ii) the value of q.

(b) A triangular lamina with vertices A, B and C has the portion inside its incircle (the circle that touches the three sides of the triangle) removed. D is the centre of the incircle. The co-ordinates of the points are $A(0,\ 0)$, $B(0,\ 27)$, $C(36,\ 0)$ and $D(9,\ 9)$.



Find the co-ordinates of the centre of gravity of the remaining lamina.

(a)
$$p = \frac{4(p)+7(-6)+3(9)+5(12)}{19}$$
$$p = 3$$
$$q = \frac{4(2)+7(1)+3(q)+5(13)}{19}$$
$$q = 5$$

5
5
5
5

(b)

	area:	c.g.
ABC	$\frac{1}{2}(36)(27) = 486$	$(12, 9)$
circle	$\frac{22}{7}(9)^2 = 254{\cdot}57$	$(9, 9)$
lamina	$= 231{\cdot}43$	(x, y)

5
5
5

$$(231{\cdot}43)(x) = 486(12) - 254{\cdot}57(9)$$
$$x = 15{\cdot}3$$

10

$$(231{\cdot}43)(y) = 486(9) - 254{\cdot}57(9)$$
$$y = 9$$

5

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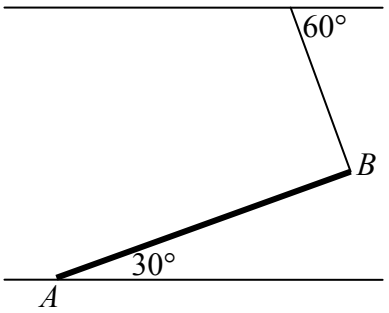
7. A uniform rod, $[AB]$, of length 4 m and weight 80 N is smoothly hinged at end A to a horizontal floor.

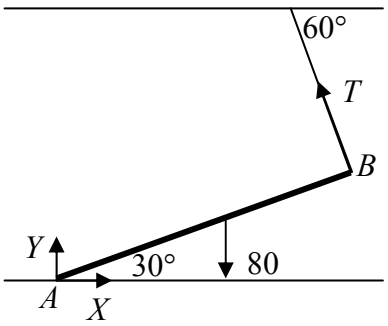
One end of a light inelastic string is attached to B and the other end of the string is attached to a horizontal ceiling.



The string makes an angle of $60°$ with the ceiling and the rod makes an angle of $30°$ with the floor, as shown in the diagram.
The rod is in equilibrium.

(i) Show on a diagram all the forces acting on the rod $[AB]$.
(ii) Write down the two equations that arise from resolving the forces horizontally and vertically.
(iii) Write down the equation that arises from taking moments about the point A.
(iv) Find the tension in the string.
(v) Find the magnitude of the reaction at the hinge, A.

(i)



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(ii) $X = T\cos 60$ 5

$Y + T\sin 60 = 80$ 5

(iii) $T \times 4 = 80 \times 2\cos 30$ 10

(iv) $T \times 4 = 80 \times 2\cos 30$

$T = 20\sqrt{3}$ 10

(v) $X = T\cos 60 = 10\sqrt{3}$

$Y = 80 - T\sin 60 = 50$

$R = \sqrt{\left(10\sqrt{3}\right)^2 + 50^2}$

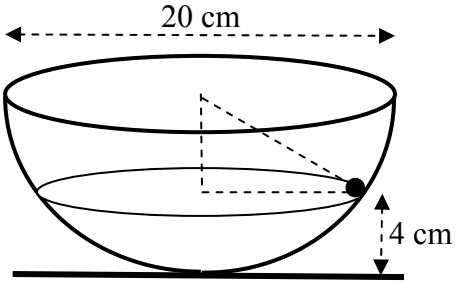
$= 20\sqrt{7}$ 10

50

8. **(a)** A particle describes a horizontal circle of radius 2 metres with uniform angular velocity ω radians per second.
Its speed is 6 m s^{-1} and its mass is 4 kg.

Find **(i)** the value of ω
(ii) the centripetal force on the particle.

(b) A hemispherical bowl of diameter 20 cm is fixed to a horizontal surface.



A smooth particle of mass 1 kg describes a horizontal circle of radius r cm on the smooth inside surface of the bowl.

The plane of the circular motion is 4 cm above the horizontal surface.
Find **(i)** the value of r
(ii) the reaction force between the particle and the surface of the bowl
(iii) the angular velocity of the particle.

(a)

(i) $v = r\omega$
$6 = 2\omega$
$\Rightarrow \omega = 3$ — 10

(ii) $F = mr\omega^2$
$= 4 \times 2 \times 3^2$
$= 72$ N — 10

(b)

(i) $r = \sqrt{10^2 - 6^2}$
$= 8$ cm — 10

(ii) $R\sin\alpha = 1g$
$R \times \frac{6}{10} = 10 \quad \Rightarrow R = 16{\cdot}7$ N — 10

(iii) $R\cos\alpha = mr\omega^2$
$\frac{100}{6} \times \frac{8}{10} = 1 \times 0{\cdot}08 \times \omega^2$
$\omega = 12{\cdot}9$ — 10

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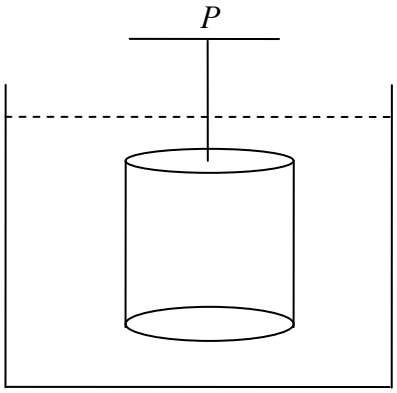
9. **(a)** State the Principle of Archimedes.
A solid piece of metal has a weight of 26 N.
When it is completely immersed in water the metal weighs 21 N.
Find **(i)** the volume of the metal
(ii) the relative density of the metal.

(b) A right circular solid cylinder has a base of radius 8 cm and a height of 18 cm.



The relative density of the cylinder is 3 and it is completely immersed in a tank of liquid of relative density 0·9.

The cylinder is held at rest by a light inextensible vertical string which is attached to a fixed point P.
The upper surface of the cylinder is horizontal.
Find the tension in the string.
[Density of water = 1000 kg m^{-3}].

(a)

Principle of Archimedes — 5

(i) $B = \rho V g$
$5 = 1000 \times V \times 10$
$V = 0{\cdot}0005 \text{ m}^{-3}$ — 10

(ii) $26 = W$
$26 = \rho V g$
$26 = 1000s \times 0{\cdot}0005 \times 10$
$s = 5{\cdot}2$ — 10

(b)

$B = 900\left\{\pi \times (0{\cdot}08)^2 \times 0{\cdot}18\right\}(10)$ — 10
$= 10{\cdot}368\pi$
$W = 3000\left\{\pi \times (0{\cdot}08)^2 \times (0{\cdot}18)\right\}(10)$ — 10
$= 34{\cdot}56\pi$

$T + B = W$
$T = 34{\cdot}56\pi - 10{\cdot}368\pi$ — 5
$= 24{\cdot}192\pi = 76$ N

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